

ATUL VIDYALAYA FIRST PRELIMINARY EXAMINATION-2012-13 MATHEMATICS

MM: 100 TIME: 3 hrs

GENERAL INSTRUCTION

(Three hours)

(Candidate are allowed additional 15 minutes for **only** reading the paper . They must **NOT** start writing during this time)

There will be one paper of **three** hours duration of 100 marks. The syllabus is divided into three sections A, B and C. Section A is compulsory for all candidates. Candidates will have choice of attempting questions from **either** from Section B or Section C.

Section A(80 marks) will consists of 9 questions. Candidate will be required to answer **Question -1** (Compulsory) and five out of the rest of the eight question.

Section B/C(20 marks) Candidate will be required to answer two questions out of three from either Section B or Section C. Section – A

Question 1.

ii)

iv)

vi)

i) Find the equation of the ellipse whose foci are are at the point $(\pm 2, 0)$ and whose latus rectum is 6. [3]

$$\frac{dy}{dx} = \sqrt{y - x}$$
[3]

iii) Evaluate : $\int \sin^3 x \cos^2 x dx$. [3]

Solve the diffential equation :

$$\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}, \text{ show that } xy + yz + zx = 1.$$
 [3]

v) Evaluate :
$$\int_{1}^{2} \frac{2dx}{4x^{2}-1}$$
. [3]

Prove that:
$$\left(\frac{\sqrt{3}+i}{2}\right)^6 + \left(\frac{i-\sqrt{3}}{2}\right)^6 = -2$$
 [3]

vii) Evaluate :
$$\lim_{x\to 0} \frac{\log(1+x)}{\log(1+x)}$$
[3]

iix) Verify Rolle's theorem for the function $f(x) = x^2 + x - 6$ in the interval [-3, 2]. [3] ix) Two dice are drawn together .What is the probability that the sum of the numbers on the two faces is greater than 8? [3]

x) Solve the following differential equation : $x \cos y dy = (xe^x \log x + e^x) dx$ [3]

Question 2

i) Prove that
$$\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz$$
i)
$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}, \text{ find } A^{-1} \text{ and hence solve the system of linear equations}$$
[5]

x + 2y - 3z = -4
2x + 3y + 2z = 2
3x - 3y - 4z = 11

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Question 3.

i)

Evaluate :
$$\int e^x \left(\frac{2+\sin 2x}{1+\cos 2x}\right) dx$$

Show that the line $\sqrt{3}y = x + 3$ touches the ellipse $2x^2 + 3y^2 = 6$. Also, find the point ii) of contact. [5]

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Question 4.

- A and B throw a coin alternately till one of them gets a "head " and wins the game. i) Find their respective probabilities of winning . [5]
- A bag contains 5 green and 7 red balls .Two balls are drawn . What is the probability ii) that one is green and other is red? [5]

Question 5.

- Evaluate : $\int (\sin^{-1} x)^2 dx$ i) [5]
- Use Boolean algebra to simplify the expression pq(r+p')+q(q'+r') and construct ii) the simplified circuit . [5]

Question 6.

i)	Show that the area enclosed between the curves $y^2 = 4ax$ and $x^2 = 4ay$ is	
	$\frac{16}{a^2}$	
	³ sq. units.	[5]

If 2x + 3y = 4, find the maximum or minimum value of xy. ii) [5]

Question 7.

Find the area enclosed by the curves $y^2 = x$ and $y^2 = 4 - 3x$. i) [5]

 $(1 \rightarrow 1 \cdot (1 \rightarrow 1))$

If
$$y = a \cos(\log x) + b \sin(\log x)$$
, where a, b are constants, show that

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

Question 8.

- The equations of two regression lines obtained in a correlation analysis are as i) follows : 2x+3y-10=0 and 4x+y-5=0. Find (i) the means of x and y.
 - (ii) regression coefficient s b_{yx} and b_{xy} (iii) the correlation coefficient. [5]
- The marks secured by a group of 10 students in the written selection test (X) and ii) In the aptitude Test (Y) are given below :

Х	44	42	40	52	39	32	24	46	41	50
Y	24	25	28	29	32	35	36	41	45	50

[5]

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[5]

[5]

Calculate the product moment correlation coefficient and rank correlation coefficient . Why do the two coefficient differ ? [5]

Question 9

ii)

- i) Solve: $\frac{dy}{dx} + \frac{1 + \cos 2y}{1 \cos 2x} = 0$. [3]
 - For what real values of x and y, $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i?$
- iii) If 1, ω , ω^2 are the cube roots of unity, prove that

$$(2+5\omega+2\omega^{2})^{6} + (2+2\omega+5\omega^{2})^{6} = 729$$
[2]

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[2]

[3]

iv) Solve :
$$x \log x \frac{dy}{dx} + y = 2 \log x$$

(Answer two question from either Section B or Section C) SECTION B

Question 10

- i) If $a = 7\hat{i} 2\hat{j} + \hat{k}, b = \hat{i} 2\hat{j} + 2\hat{k}, c = 3\hat{i} 8\hat{j}$, then find a.(bXc) and (aXb).c. Is a.(bXc) = (aXb).c? [5]
- Show by vector method that the parallelogram whose diagonals are equal , is a rectangle .

Question 11.

- i) Find the equation of the planes through the intersection of the planes x+3y+6=0and 3x-y-4z=0 whose perpendicular distance from the origin is equal to 1. [5]
- ii) Find the equation of the line passing through the point (1,1,-3) and perpendicular

to the lines
$$\frac{x-2}{2} = \frac{y+3}{1} = \frac{z}{-3}$$
 and $x-3 = y = z+5$. [5]

Question 12.

- i) By examining the chest X-ray, probability that T.B. is detected when a person is actually suffering is 0.99. The probability that the doctor diagnoses incorrectly that a person has T.B. on the basis of X-ray is 0.001. In a certain city 1 in 1000 persons suffers from T.B. A person is selected at random is diagnosed to have T.B. What is the chance that he actually has T.B.?
- ii) Four bad oranges are mixed accidentally with 16 good oranges . Find the probability Distribution of the number of bad oranges in a draw of two oranges . [5]

SECTION C

Question 13.

- The bankers discount and banker's gain on a certain bill of exchange due after certain time are , respectively , `1250 and `50 . Find the face value of the bill. [5]
- The price of a radio is ` 500 cash as it may be paid for by 5 equal monthly
 Installments of ` 110 each , the first installment to be paid one month after purchase .
 Find the rate of interest charged .

Question 14

- A loan of `10,000 is repaid at 5% per annum compound interest in 12 equal installments beginning at the end of first year. Find the amount of each installment.
- ii) The demand function for a certain product is represented by the following equation: $p = 20 + 5x - 3x^2$, where x is the number of units demanded and p is the price per units.
 - a) Find the marginal revenue .
 - b) Obtain the marginal revenue when 2 units are sold.

Question 15.

i) Find by A.M. method , the index number from the following data :[5]

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[5]

[5]

Commodity	Base Price	Current Price	Weight		
Rice	30	52	8		
Wheat	25	30	6		
Fish	130	150	3		
Potato	35	49	5		
Oil	70	105	7		

ii) Obtain the five-year moving averages for the following data :

Year	1960	1961	1962	1963	1964	1965	1966	1967	196
									8
Annua	36	43	43	34	44	54	34	24	14
I Sales									

Construct also the 4 year centred moving averages .

[5]

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